COGNITIVE MOTIVATION AS A CRUCIAL FACTOR IN MATHEMATICS EDUCATION

D.B. Bogoyavlenskaya, A.N. Nizovtsova

Russian Academy of Education Psychological Institute, Russian Federation

The article relates the leading determinants of giftedness to the specific example of mathematical giftedness. Students and post-graduates of mathematical specialities at prestigious Moscow universities (n=100) are the participants of the current research. Giftedness is diagnosed with a technique designed in the frame of the Creative Field Method using mathematical material. The findings are compared with the results of techniques that examine intellect and personality. It has been demonstrated that the general intellect correlates to successfully mastering mathematical activity, but it cannot definitely predict mathematical giftedness. The manifestation of the latter depends on cognitive motivation and the so-called "worldview activeness".

INTRODUCTION

Identifying and supporting children with general and special giftedness is a topical issue at the state level in many countries. For the most part, the education of gifted children is carried out according to formal criteria, such as school progress and winning various contests. Those circumstances call for a scientifically justified definition of giftedness.

THEORETICAL BACKGROUND

When diagnosing giftedness, the established tradition is to reduce it to a high level of ability as demonstrated in the work of Binet, Eysenck, Terman, Spearman, Wechsler, Stern. However, the results of modern research show the impossibility of explaining the achievements of gifted children and adults by the specifics of their intellect. Numerous works cover various traits of a gifted person and his/her motivation (Bogoyavlenskaya, 2011, Melik-Pashaev, 2018; Renzulli,1984; Heller et al, 2005; Csikszentmihalyi, 1997.). Nevertheless, according to Panov, a peculiar "diagnostics paradox" can be observed: it is stated in psychology that giftedness should not be reduced to the level of ability, and yet it is diagnosed, as a rule, by evaluation of various abilities (Panov, 2014).

This contradiction can be seen with utmost clarity in the example of mathematical giftedness: the scientific literature shows that it is defined in most cases based on individual abilities in mathematical processes (Krutetzkii 1969; Yakimanskaya, 2004; Presmeg, 2006).

2019. In NNN (Eds.). *Proceedings of the PME & Yandex Russian Conference*, pp. XX-YY. Moscow, Russia: PME.

Meanwhile, some studies of mathematical giftedness cover the role of personality traits in its structure. Goldin supposes that the student's affective system occupies a central place in his/her cognition and its influence can either raise or lower cognitive activity (Goldin, 2002). Jensen mentions that schoolchildren who are oriented at a problem continue to solve it even when difficulties arise. Those who do not have a considerable interest in the problem make an effort only as necessary not to fail (Jensen, 1973). Bargdill and Starko discuss the role of the internal motivation for creativity development as well: the higher the level of a child's internal motivation, the more probable are creative solutions and discoveries (Bargdill & Starko, 2006). McLeod finds a positive correlation between the attitude toward a problem and achievements in various classes (McLeod, 1992). Plucker and Renzulli suppose that a positive attitude to the subject can be considered as an index of creative potential (Plucker & Renzulli, 1999).

According to our theory of giftedness and creativity (Bogoyavlenskaya, 1971, 2011), at the stage of mastering an activity a corresponding level of the general intellect is necessary, but the future performance of the activity is determined by the person's system of motives and values. One person might simply solve the problem. Another one, infatuated by the process during problem solving becomes absorbed, considers the activity more widely, and beyond reaching the initial goal discovers new regularities. It is the development of the activity through one's initiative that is considered as a unit of creativity and characterizes a personality, in whose structure cognitive motivation dominates. Yet giftedness is defined as the ability to demonstrate creativity – developing the activity by one's initiative. This approach has not been applied previously to the topic of mathematical giftedness.

International studies of the specifics of mathematical giftedness in the context of problem correlation of general, special and creative abilities is a relatively recent phenomenon (Hong & Aqui, 2004; Kattou et al., 2013; Kontoyianni et al., 2013; Leikin & Pitta-Pantazi, 2013; Pitta-Pantazi et al., 2011; Sriraman et al., 2013), whereas in the Soviet/ Russian psychology field the correlation between various kinds of abilities has been studied since the second half of the 20th century (Rubinstein, 1960; Teplov, 1961; Krutetskii, 1969). In Shadrikov's theory (Shadrikov, 2010), abilities are understood as characteristics of functional systems: special abilities are considered to be general ones which have acquired the characteristic of responsiveness under the influence of the demands of the activity. Thus, the contradiction dissolves and the issue of the nature of special abilities gets a definite answer. Abilities are considered at three levels: the level of the individual (natural abilities); the level of the activity subject (special abilities) and the level of the personality (including his/her moral sphere). In Shadrikov's opinion, abilities at the level of the personality represent giftedness, which can transfer to creativity.

EMPIRICAL RESEARCH (the data are presented by Nizovtsova)

The research aim is the analysis of the psychological structure of mathematical giftedness and defining its cognitive and personal components.

RESEARCH HYPOTHESES

1. Successfully mastering mathematical activity should be connected with the level of the general intellect, but a high intellectual level without considering personality traits does not definitely predict mathematical giftedness.

2. The manifestation of mathematical giftedness should depend on the dominating cognitive motivation of the personality (which can be seen in constructive motivation and infatuation with the subject) and the so-called “worldview activeness”.

RESEARCH METHOD

The Creative Field Method allows diagnosing such personality trait as the Intellectual Initiative (II) - the development of an activity by one’s initiative (extending beyond the limits of the given task). The method defines three levels of work: (1) successfully mastering the given activity (stimulus-productive level); (2) the ability to develop the activity by one’s initiative, which allows discovering new regularities (heuristic level); and (3) proving the discovered regularities (creative level). In the frame of this method, the Coordinate System technique, using mathematical material, has been designed and applied by researchers (Bogoyavlenskaya, 1971, 2011; Petukhova, 1976).

The advanced Raven’s Progressive Matrices (Raven, 2002) is a non-verbal intellect test designed for a finer differentiation when the participant’s abilities exceed the medium level. The test includes two series, the first containing twelve tasks and the second comprised of thirty-six tasks.

The diagnostics of the motivational structure of personality (Milman, 2005) describes the following motivational scales reflecting the main personality orientations: consumer tendency (motivation of life support, motives of comfort and safety, status and prestige motivation) and constructive tendency (motivation of general activeness, motivation of creative activeness, motivation of the public benefit). Each scale is divided into two subscales: the ideal state of the motive and its real state.

The technique of evaluating worldview activeness (Leontiev & Ilchenko, 2007) includes 13 pairs of statements referring to various aspects of human life. Each pair has the same beginning and two variants of an ending (A and B). The participant is asked to evaluate the degree of his/her concordance with the variant of the statement from 0 to 100% (0 - totally disagree, 100% - completely agree). The instruction states that the sum does need to be 100%. A participant can also introduce his/her own variants of the answer if not satisfied with those provided. There are 4 types of answers: 1) definite answers (A=100%, B=0% or vice versa), 2) combinations of two variants (A + B ≤100%), 3) crossing responses (A + B > 100%) and 4) self-contained answers. The first two types of answers indicate the worldview passivity of a person, expressed in the uncritical acceptance of another's opinion. The answers of the third type indicate the worldview multidimensionality, such person has a broader view of the world, assumes that there are hidden dimensions in the world. The answers of the fourth type are attributed to the highest type of worldview activeness, such people clearly have a desire for change, tolerance for uncertainty and a sense of meaningfulness of their lives.

RESEARCH PARTICIPANTS

Between 2011 and 2017, 100 participants (68 males and 32 females; ages 18 to 34 years, M = 23,55, σ = 2,76) took part in the research. Forty-two of them were students of mathematical specialities at Moscow universities (MSU, MIPT, MEPHI, BMSTU), 15 were PhD students and 43 were mathematicians and programmers, eleven having PhDs in physical-mathematical sciences.

RESEARCH RESULTS AND DISCUSSION

As the result of the Coordinate System Technique, the following distribution according to the II levels has been obtained: 72 people at the stimulus-productive level; 24 people at the heuristic level; and 4 at the creative level. For the purposes of statistical processing, the participants at the creative and heuristic levels were united in the same group because of the small number of the former.

The special feature of the Creative Field Method is that it allows diagnosing both the intellectual initiative (extending beyond the limits of the given task) and the general level of mental capabilities (in the frame of mastering the given layout of the activity). When comparing the results of the Raven's Test and the indices of educability in the Coordinate System Technique, correlations with the time to solve problem are obtained ($r = -0.35$, $p<0.001$). There are no correlations between the results of the Raven's Test and the indices of the intellectual initiative in the Coordinate System Technique. The analysis of the results of the two groups of participants shows that those at the stimulus-productive level do not differ in the Raven's Test results from the participants at the heuristic level ($t = -0.65$, $p = 0.52$). Thus, based on the obtained results, it is possible to conclude that it would not be correct to rely on high scores in intellectual tests as the only sufficient criterion for the identification of giftedness.

The application of Milman's technique shows that, in the ideal plan, participants at heuristic and stimulus-productive levels have similar results. The only difference is in the motivation of comfort and safety: in the ideal plan, the motivation of comfort for the heuristic level is less important than for the participants at the stimulus-productive level ($t = 4.36$, $p<0.001$). The main differences can be seen in the participants' real motivation. Thus, those at heuristic and stimulus-productive levels differ in the motivation of general activeness ($t = -4.04$, $p<0,001$), motivation of creative activeness ($t = -5.90$, $p<0,001$) and motivation of the public benefit ($t = -3.82$, $p<0.001$). According to Milman, those three kinds of motivation form a constructive tendency of the personality. In the open questions of the questionnaire, the participants at the heuristic level describe real situations from their lives confirming their answer choices. Thus, one participant at the heuristic level became the youngest member of the jury of the All-Russian Olympiad of Schoolchildren in Mathematics. Another participant at the heuristic level is now a Chair of the Organizing Committee of the Tournament of the Cities in Moscow and one of three members of the Central Organizing Committee with a casting vote.

According to Leontiev, creative synthesis is defined as finding a new solution by revision of the question's statement, from which comes an incompatibility of the experience elements. In this case, the maximum degree of "worldview activeness" is demanded from the participant (Leontiev & Ilchenko, 2007). It can be rightfully brought into correlation with intellectual initiative (see Bogoyavlenskaya, 2011) as it relates to the going beyond the limits of the given activity as well.

The data obtained in the Technique of Worldview Activeness were recoded from two possible variants into four. There were 19.5% definite answers (A=100%, B=0% or vice versa), 29.1% combinations of two variants ($A + B \leq 100\%$), 16.4% crossing responses ($A + B > 100\%$) and 35% self-contained answers. Meanwhile, the participants of the heuristic and stimulus-productive levels differed significantly in the distribution of the frequencies of various answer types ($\chi^2 = 59.579$, df = 3, $p<0.001$). According to the results of the correlation analysis of the frequencies of various answer types in the Technique of Worldview Activeness and main indices of the Coordinate System Technique, a correlation between the frequency of the self-contained answers and manifestations of the intellectual initiative (r = 0,545, $p<0.001$) has been found. There is also an inverse correlation of intellectual initiative and the frequency of definite answers ($r = -0.453$, $p<0.001$) and the combination of two variants ($r = -0.219$, $p = 0.029$).

To define the interconnection between cognitive and personality components, we carried out a logistic regression analysis. Belonging to the stimulus-productive or heuristic level categories of the II, manifestation of mathematical giftedness is considered as the dependent variable. For the predictors at the first stage, we introduced personality components: worldview activeness and constructive motivation. At the second stage, a cognitive component (Raven's general intellect index) was added. The prognosis of the regression Model 1, including only personality components, proved correct for 88% of the participants. When adding the cognitive component (Raven's general intellect index) into Model 2, there were no sufficient changes. Based on the obtained results, we accepted Model 1, which shows that worldview activeness ($\beta = 1.15$, $p<0.001$) and constructive motivation ($\beta = 0.455$, $p = 0.001$) are the significant predictors of mathematical giftedness. General intellect according to the obtained data does not predict the manifestation of mathematical giftedness.

Without knowing it, the research participants gave very precise definitions of how various qualities relate to each other in the structure of mathematical giftedness.

Participant #16 (stimulus-productive level): "Well, kindness or something else… are not the qualities to define or not to define a mathematician. The matter is, that what can help in the work is needed: diligence, persistence, intuition. If you are talented and diligent, that is enough."

Participant #69 (heuristic level): "In my opinion, interest, motivation and love for mathematics – that is all that needed. Interest is quite enough. If a person is really interested in something certain, then he can achieve everything no matter what he had in the beginning."

Participant #98 (heuristic level): "In any case there should be a passion for solving the problem, this eagerness to disclose it, to find the unknown. And if there is such eagerness, then to master a set of tools or scientific materials, maybe a language to read other articles, to communicate with colleagues from other countries will be no problem. So, the main thing is passion."

Those fragments illustrate the differences between participants of the stimulus-productive and heuristic levels. If the former talk about the things which are "important and needed for work", then the latter prioritize interest and "passion", and the other qualities by this attitude become just "instruments".

CONCLUSION

In the current work, the theory of giftedness and creativity (Bogoyavlenskaya, 1971, 2011) has been applied to the topic of special giftedness using the example of mathematical giftedness. According to this theory, mathematical giftedness is understood as the ability to develop an activity by one's initiative in the sphere of mathematics and is discovered as a system quality, integrating cognitive and personality components when cognitive motivation dominates in the personality structure. The participants of the current research were not random, it was students and post-graduates of mathematical specialities at prestigious Moscow universities, each one achieved a success in mathematics. The study is shown that the general intellect (on the advanced Raven's Progressive Matrices) is connected with mastering mathematical activity (on the Coordinate System technique), but it cannot definitely predict mathematical giftedness. Its manifestation depends on the cognitive orientation of the personality (which appears as constructive motivation for and infatuation with the subject matter) and the so-called "worldview activeness". Thus, based on the obtained results, it is possible to conclude that it would not be correct to rely on high scores in intellectual tests as the only sufficient criterion for the identification of giftedness The teaching mathematics should consist of not only disseminating certain knowledge, but of forming by students cognitive motivation (as internal process) in the first place.

References

Bargdill R.W., & Starko, A. J. (2006). Creativity in the classroom: Schools of curious delight. *Journal of Phenomenological Psychology, 1*, 124-128.

Bogoyavlenskaya, D. B. (1971). Metod issledovaniya urovnei intellektual'noi aktivnosti [The study method of the levels of intellectual activity]. *Voprosy Psikhologii, 1*, 144–146. (in Russian)

Bogoyavlenskaya, D. B. (2011). On Prognostics of Psychodiagnostics Method Creative Field. *Psychology in Russia: State of the Art, 4*, 39-52.

Csikszentmihalyi, M. (1997). *Creativity: Flow and the psychology of discovery and invention*. New York: HarperPerennial.

Goldin G. A. (2002). Affect, meta-affect, and mathematical belief structures. In: Leder G.C., Pehkonen E., Törner G. (eds) *Beliefs: A hidden variable in mathematics education?*, 59-72. Dordrecht: Springer.

Heller, K. A., Perleth, C., & Lim, T. K. (2005). The Munich model of giftedness designed to identify and promote gifted students. *Conceptions of Giftedness, 2*, 147-170.

Hong, E., & Aqui, Y. (2004). Cognitive and motivational characteristics of adolescents gifted in mathematics: Comparisons among students with different types of giftedness. *Gifted Child Quarterly, 48*(3), 191–201.

Jensen, L. R. (1973) The relationships among mathematical creativity, numerical aptitude and mathematical achievement. *Dissertation Abstract International*, *34*(5), 2168.

Kattou, M., Kontoyianni, K., Pitta-Pantazi, D., & Christou, C. (2013). Connecting mathematical creativity to mathematical ability. *ZDM – International Journal on Mathematics Education, 45*(2), 167–181.

Kontoyianni, K., Kattou, M., Pitta-Pantazi, D., & Christou, C. (2013). Integrating mathematical abilities and creativity in the assessment of mathematical giftedness. *Psychological Test and Assessment Modeling, 55*(3), 289–315.

Krutetskii, V. A. (1969). An analysis of the individual structure of mathematical abilities in schoolchildren. *Soviet studies in the psychology of learning and teaching mathematics, 2,* 59-104.

Leikin, R., & Pitta-Pantazi, D. (2013). Creativity and mathematics education: The state of the art. *ZDM– International Journal on Mathematics Education, 45*(2), 159–166.

Leontiev, D. A., & Ilchenko, A. N. (2007). Urovni mirovozzrencheskoi aktivnosti i ikh diagnostika [The levels of worldview's activity and their diagnostics]. *Psikhologicheskaya Diagnostika, 3*, 3–21. (in Russian)

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. *Handbook of research on mathematics teaching and learning.* New York: Macmillan Publishing Co, Inc.

Melik-Pashaev, A. A. (2018). O hudozhestvennoj odarennosti i odarennosti kak takovoj [About artistic giftedness and giftedness as such] In D. B. Bogoyavlenskaya, V. K. Baltyan (Eds.), *Proc. All-Russia Conference "Giftedness: methods of identifying and ways of development"* (Vol. 1, pp. 20-27). Moscow: Bauman MSTU. (in Russian)

Milman, V. E. (2005). *Motivatsiya tvorchestva i rosta. Struktura. Diagnostika. Razvitie* [Motivation of creativity and growth: Structure. Diagnostics. Development]. Moscow: OOO "Mireya i Ko". (in Russian)

Panov V. I. (2014). Odarennost': ot paradoksov k razvitiyu sub"ektnosti [Giftedness: from paradoxes to development of subjectivity]. *Izvestiya MGTU «MAMI», 4*, 129-137. (in Russian)

Petukhova, I. A. (1976). Umstvennye sposobnosti kak component intellektual'noi initsiativy [Intellectual abilities as a component of intellectual initiative]. *Voprosy Psikhologii*, 4, 80–89. (in Russian)

Plucker J. A., & Renzulli J. S. (1999). Psychometric approaches to the study of human creativity. R. J. Sternberg (Ed.), *Handbook of creativity*, 35-61. Cambridge: Cambridge University Press.

Presmeg, N. C. (2006). Research on visualization in learning and teaching mathematics. *Handbook of research on the psychology of mathematics education,* 205-236. Rotterdam: Sense Publisher.

Pitta-Pantazi, D., Christou, C., Kontoyianni, K., & Kattou, M. (2011). A model of mathematical giftedness: Integrating natural, creative, and mathematical abilities. *Canadian Journal of Science, Mathematics and Technology Education, 11*(1), 39–54.

Raven, J. C. (2002). *Prodvinutye progressivnye matritsy: Serii 1, 2* [Advanced Progressive Matrices: Series 1, 2]. Moscow: Kogito-Tsentr. (in Russian)

Renzulli, J. S. (1984). *The Three Ring Conception of Giftedness: A Developmental Model for Creative Productivity*. Storrs, CT: The University of Connecticut.

Rubinstein, S. L. (1960). *Problema sposobnostei i voprosy psikhologicheskoi teorii* [The problem of abilities and issues of psychological theory]. *Voprosy Psikhologii, 3,* 3–8. (in Russian)

Shadrikov, V. D. (2010). *Professional'nye sposobnosti* [Professional abilities]. Moscow: Universitetskaya Kniga. (in Russian)

Sriraman, B., Haavold, P., & Lee, K. (2013). Mathematical creativity and giftedness: a commentary on and review of theory, new operational views, and ways forward. *ZDM – International Journal on Mathematics Education, 45*(2), 215–225.

Teplov, B. M. (1961). *Problemy individual'nykh razlichii* [Problems of individual differences]. Moscow: Academy of Pedagogical Sciences of the RSFSR. (in Russian)

Yakimanskaya, I. S. (2004). *Psikhologicheskie osnovy matematicheskogo obrazovaniya* [Psychological foundations of mathematical education]. Moscow: Akademiya. (in Russian)